Charge fluctuations in the edge states of normal-superconducting hybrid nano-structures

1999 J. Phys.: Condens. Matter 11 L563

(http://iopscience.iop.org/0953-8984/11/50/102)

J. Phys.: Condens. Matter **11** (1999) L563–L568. Printed in the UK PII: S0953-8984(99)08491-X

LETTER TO THE EDITOR

Charge fluctuations in the edge states of normal–superconducting hybrid nano-structures

Andrew M Martin, Thomas Gramespacher and Markus Büttiker

Département de Physique Théorique, Université de Genève, CH-1211 Genève 4, Switzerland

Received 6 October 1999

Abstract. In this work we show how to calculate the equilibrium and non-equilibrium charge fluctuations in a gated normal mesoscopic conductor which is attached to one normal lead and one superconducting lead. We then consider an example where the structure is placed in a high magnetic field, such that the transport is dominated by edge states. We calculate the equilibrium and non-equilibrium charge fluctuations in the gate, for a single edge state, comparing our results to those for the same system, but with two normal leads. We then consider the specific example of a quantum point contact and calculate the charge fluctuations in the gate for more than one edge state.

1. Introduction

Recently a methodology for calculating charge fluctuations in gated hybrid normal–superconducting (N–S) systems has been formulated [1]. In this letter we implement this methodology for one particularly relevant and enlightening example. We study the system depicted in figure 1, i.e. a gated N–S system in a high magnetic field, where the transport in the normal region is governed by edge states. We focus on calculating the charge fluctuations in the edge states in the region Ω. This work coincides with recent experimental and theoretical research into the properties of hybrid normal–superconducting structures in high magnetic fields [2–5].

In this letter we first calculate the unscreened charge fluctuations for a general N–S structure (section 2). Then we proceed to incorporate screening into the problem (section 3), enabling us to calculate the spectra of charge fluctuations in a gated N–S structure. We show that these spectra, at equilibrium, are determined by two quantities: the charge relaxation resistance, R_q, and the electrochemical capacitance C_μ. In the presence of transport, we find that R_q is replaced by R_V, called here the Schottky resistance, which reflects the shot noise of the structure. Then we focus on one particular example (section 4) an N–S structure in a high magnetic field, see figure 1. In particular we are interested in the charge fluctuations in the edge states in the region Ω of figure 1 which can be measured by observing the current fluctuations at the gate $S_{II}(\omega, V) = \omega^2 S_{QQ}(\omega, V)$. We consider a general case of a single edge state and an arbitrary scatterer and more than one edge state in the case where the scatterer is a quantum point contact.

0953-8984/99/500563+06$30.00

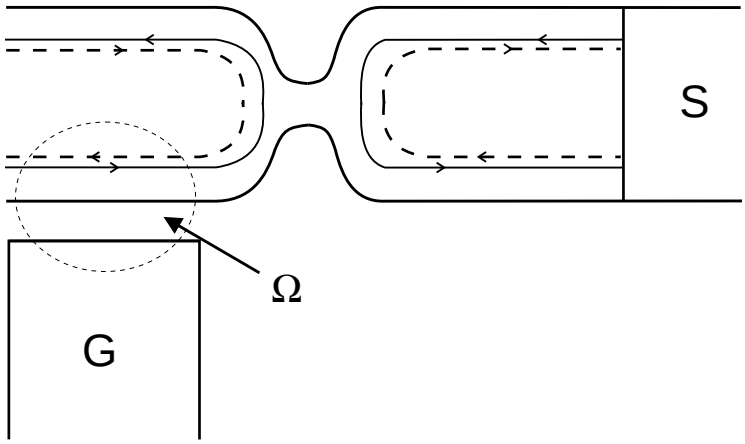

Figure 1. Quantum point contact, in the presence of a high magnetic field, attached to one normal lead and a superconducting lead, capacitively coupled to a macroscopic gate. The solid (dashed) lines depict the *motion* of the electron (hole) edge states.

2. Calculating the unscreened charge fluctuations

Consider a conductor where we have one normal lead and one superconducting lead, we can write an effective 2×2 scattering matrix for this system,

$$\hat{S} = \begin{pmatrix} s_{pp} & s_{hp} \\ s_{ph} & s_{hh} \end{pmatrix} \tag{1}$$

where the $s_{\mu\lambda}$ are elements of the scattering matrix describing the process of an electron ($\lambda = p$) or hole ($\lambda = h$) entering from the normal lead and an electron ($\mu = p$) or hole ($\mu = h$) returning to the normal lead. Making use of this $\hat{S}$-matrix [1, 6] we can write down the local particle density of states elements at position $\boldsymbol{r}$ as

$$\mathcal{N}^{\eta}_{\alpha\beta}(\lambda, \boldsymbol{r}) = \frac{-1}{4\pi i}\left[s^{\dagger}_{\lambda\alpha}(E, \underline{U}(\boldsymbol{r}))\frac{\partial s_{\lambda\beta}(E, \underline{U}(\boldsymbol{r}))}{q^{\eta}\partial U^{\eta}(\boldsymbol{r})} - s_{\lambda\alpha}(E, \underline{U}(\boldsymbol{r}))\frac{\partial s^{\dagger}_{\lambda\beta}(E, \underline{U}(\boldsymbol{r}))}{q^{\eta}\partial U^{\eta}(\boldsymbol{r})}\right] \tag{2}$$

where α, β, λ and η denote the electron/hole degrees of freedom (p/h) and $q^{p} = e = -q^{h}$. The functional derivatives are taken at the equilibrium electrostatic potential $U^{p} = U^{h} = U_{eq}$. To give an example, $\mathcal{N}^{h}_{ph}(h, \boldsymbol{r})$ is the hole density associated with an electron and a hole current amplitude incident from the normal lead and reflected back into the normal lead as an outgoing hole amplitude. With the help of this basic expression we can now find both the average density of states as well as the fluctuations. The total bare charge fluctuations in the conductor are given by

$$\mathcal{N}_{\alpha\beta} = -\mathcal{N}^{h}_{\alpha\beta} + \mathcal{N}^{p}_{\alpha\beta} \tag{3}$$

where

$$\mathcal{N}^{\eta}_{\alpha\beta} = \int_{\Omega} \mathrm{d}^{3}\boldsymbol{r} \sum_{\lambda} \mathcal{N}^{\eta}_{\alpha\beta}(\lambda, \boldsymbol{r}). \tag{4}$$

The hole density of states of a region Ω of the conductor is $N^{h} = N^{h}(p) + N^{h}(h)$ where

$$N^{h}(\alpha) = \sum_{\lambda} \int_{\Omega} \mathrm{d}^{3}\boldsymbol{r}\,\mathrm{Tr}[\mathcal{N}^{h}_{\alpha\alpha}(\lambda, \boldsymbol{r})] \tag{5}$$

and the electron density of states is $N^p = N^p(p) + N^p(h)$ with

$$N^p(\alpha) = \sum_{\lambda} \int_{\Omega} \mathrm{d}^3\boldsymbol{r}\,\mathrm{Tr}[\mathcal{N}^p_{\alpha\alpha}(\lambda, \boldsymbol{r})]. \tag{6}$$

The trace is over open quantum channels, $N^\alpha(\beta)$ is the injectivity of electrons (holes) ($\beta = p(h)$), from the normal lead into the conductor, given a change in the particle (hole) potential ($\alpha = p(h)$).

The fluctuations of the bare charge in a region Ω of interest can be found from the charge operator $e\hat{\mathcal{N}}$ given by

$$e\hat{\mathcal{N}}(\omega) = \sum_{\substack{\alpha\beta \\ \eta\lambda}} \int_{\Omega} \mathrm{d}^3\boldsymbol{r} \int \mathrm{d}E[\hat{a}^\dagger_\alpha(E) q^\eta \mathcal{N}^\eta_{\alpha\beta}(\lambda, \boldsymbol{r}; E, E + \hbar\omega)\hat{a}_\beta(E + \hbar\omega)] \tag{7}$$

where the zero-frequency limit of $\mathcal{N}^\eta_{\alpha\beta}(\lambda, \boldsymbol{r}; E, E+\hbar\omega)$ is given by equation (2). In equation (7), $\hat{a}^\dagger_\alpha(E)$ ($\hat{a}_\alpha(E)$) creates (annihilates) an incoming electron/hole ($\alpha = p/h$) in the normal lead. The true charge fluctuations must be obtained by taking into account the Coulomb interaction and below we show how to obtain the true charge fluctuations from the fluctuations of the bare charges.

3. Calculating the total charge fluctuations

Consider an N–S system where we have a gate, Coulomb coupled, to a region, Ω, of the normal part of the system. Then, assuming the gate has no dynamics of its own, the charge in the region coupled to the gate is given by $\hat{Q} = C\hat{U}$ where $\hat{U}$ is the operator for the internal potential of the conductor. Also we can determine the charge $\hat{Q}$ as the sum of the bare charge fluctuations $e\hat{\mathcal{N}}$ and the induced charges generated by the fluctuating induced electrical potential. In the random phase approximation the induced charges are proportional to the average frequency-dependent density of states $N_\Sigma(\omega)$ times the fluctuating potential. Hence, the net charge can be written as $\hat{Q} = C\hat{U} = e\hat{\mathcal{N}} - e^2 N_\Sigma(\omega)\hat{U}$, this is the direct analogue of the result for a system with two normal leads [7]. Thus

$$\hat{U} = Ge\hat{\mathcal{N}} \tag{8}$$

where

$$G(\omega) = \frac{1}{C + e^2 N_\Sigma(\omega)} \tag{9}$$

and the total density of states is

$$N_\Sigma = \tfrac{1}{2}[N^p(p) - N^p(h) + N^h(h) - N^h(p)]. \tag{10}$$

We now wish to find the fluctuation spectra of the internal potential [8]

$$2\pi S_{UU}(\omega)\delta(\omega - \omega') = (1/2)\langle \hat{U}(\omega)\hat{U}(\omega') + \hat{U}(\omega')\hat{U}(\omega)\rangle. \tag{11}$$

Solving the above and making use of the fact that $C^2 S_{UU}(\omega, V) = S_{QQ}(\omega, V)$ we find

$$S_{QQ}(\omega, V) = (1/2)C_\mu^2 N_\Sigma^{-2} \sum_{\alpha\beta} \int \mathrm{d}E\, F_{\alpha\beta}(E, E + \hbar\omega)\mathrm{Tr}[\mathcal{N}_{\alpha\beta}(E, E + \hbar\omega)\mathcal{N}^\dagger_{\alpha\beta}(E, E + \hbar\omega)] \tag{12}$$

where

$$\mathcal{N}_{\alpha\beta}(E, E') = \sum_{\eta\lambda} \mathrm{sgn}(q^\eta) \int_{\Omega} \mathrm{d}^3\boldsymbol{r}\,\mathcal{N}^\eta_{\alpha\beta}(\lambda, \boldsymbol{r}; E, E') \tag{13}$$

$$C_\mu = \frac{Ce^2 N_\Sigma}{C + e^2 N_\Sigma} \tag{14}$$

and

$$F_{\alpha\beta}(E, E') = f_\alpha(E)[1 - f_\beta(E')] + f_\beta(E')[1 - f_\alpha(E)]. \tag{15}$$

In the above the sum is over all degrees of freedom $\alpha\beta$, the Fermi functions $f_\alpha(E)$ are defined such that $f_p = f_0(E_p - \mu)$ and $f_h = f_0(E_h + \mu)$ where E_α is the energy of a particle (hole) ($\alpha = p(h)$) in the normal reservoir, which is at a chemical potential μ, $f_0(E)$ is the Fermi function at the condensate chemical potential of the superconducting lead (μ_0).

Now we wish to evaluate equation (12) at equilibrium and zero temperature, to leading order in $\overline{h}\omega$. We find

$$S_{QQ}(\omega) = 2C_\mu^2 R_q \overline{h}|\omega| \tag{16}$$

with an equilibrium charge relaxation resistance

$$R_q = \frac{h}{2e^2}\frac{\sum_{\alpha\beta}\mathrm{Tr}(\mathcal{N}_{\alpha\beta}\mathcal{N}_{\alpha\beta}^\dagger)}{[N_\Sigma]^2}. \tag{17}$$

Also we find at zero temperature to leading order in $e|V|$ that

$$S_{QQ}(V) = 2C_\mu^2 R_V e|V| \tag{18}$$

with a Schottky resistance

$$R_V = \frac{h}{2e^2}\frac{\mathrm{Tr}(\mathcal{N}_{ph}\mathcal{N}_{ph}^\dagger + \mathcal{N}_{hp}\mathcal{N}_{hp}^\dagger)}{[N_\Sigma]^2}. \tag{19}$$

4. Example

We now wish to consider the structure shown in figure 1. If there is only one edge state then we can proceed to consider the situation when the gate only *sees* the edge state in the area Ω. In this case a particle (hole) transversing the length *seen* by the gate acquires a phase ϕ_p (ϕ_h), this phase is determined by the potential in this region. This simplifies the s-matrix given by equation (1) to

$$\hat{S} = \begin{pmatrix} r_o \exp \mathrm{i}\phi_p & r_a \exp \mathrm{i}(\phi_p + \phi_h) \\ r_a^\star & r_o^\star \exp \mathrm{i}\phi_h \end{pmatrix} \tag{20}$$

where $r_o r_o^\star$ is the probability of normal reflection (R_o^{NS}) from the N–S structure, $r_a r_a^\star$ is the probability of Andreev reflection (R_a^{NS}) and $R_a^{NS} + R_o^{NS} = 1$. We can then write, for the edge state described by the above scattering matrix, $\mathrm{d}s/q^\lambda \mathrm{d}U^\lambda = (\mathrm{d}s/\mathrm{d}\phi_\lambda)(\mathrm{d}\phi_\lambda/q^\lambda \mathrm{d}U^\lambda)$ and that $(\mathrm{d}\phi_\lambda/q^\lambda \mathrm{d}U^\lambda) = -2\pi \mathrm{d}N_\lambda/\mathrm{d}E$, where $\mathrm{d}N_\lambda/\mathrm{d}E$ is the particle (hole) ($\lambda = p(h)$) density of states in the normal lead. After some algebra and assuming $\mathrm{d}N_h/\mathrm{d}E = \mathrm{d}N_p/\mathrm{d}E = N$ we can compare the charge relaxation resistance for the N–S structure (R_q^{NS}) and the same structure but with both leads being normal (R_q^N). We find $R_q^{NS} = h/(2e^2 R_o^{NS})$ and $R_q^N = h/(2e^2)$. Hence in equilibrium we find for the spectra of the charge fluctuations that

$$S_{QQ}^{NS}(\omega) = \frac{he^2 N^2 R_o^{NS} C^2}{(C + e^2 N R_o^{NS})^2}\overline{h}|\omega| \tag{21}$$

and

$$S_{QQ}^{N}(\omega) = \frac{he^2 N^2 C^2}{(C + e^2 N)^2}\overline{h}|\omega|. \tag{22}$$

In the presence of an applied voltage we find the Schottky resistances $R_V^{NS} = R_a^{NS} h/(2e^2 R_o^{NS})$ and $R_V^N = R_o^N(1 - R_o^N)h/e^2$, where $R_o^N = 1 - T_o^N$ is the probability of normal reflection when

we have replaced the superconducting lead with a normal lead. Hence the non-equilibrium charge fluctuations are given by

$$S_{QQ}^{NS}(V) = \frac{he^2N^2R_o^{NS}R_a^{NS}C^2}{(C + e^2NR_o^{NS})^2}e|V| \tag{23}$$

and

$$S_{QQ}^{N}(V) = \frac{he^2N^2T_o^{N}R_o^{N}C^2}{(C + e^2N)^2}e|V|. \tag{24}$$

The results above are for one edge state but the striking difference between the results for a N–S system as compared to a system where both leads are normal occurs when more than one edge state is present. To consider this scenario we shall focus on the particular example of our scattering region being a quantum point contact. For such a system only one quantum channel opens at a time, i.e. for all other open channels the transmission through the quantum point contact is 1, in the case of an ideal superconducting interface this implies that, for these open channels $R_a^{NS} = 1$. Hence, considering an additional edge state which has perfect transmission through the quantum point contact then this state generates no extra noise, and no extra contribution to screening, i.e. the total added charge is zero (C_μ for this state is zero), thus equations (21) and (23) remain valid, where R_o^{NS} and R_a^{NS} are the reflection probabilities for the opening quantum channel. This is in contrast to what happens to $S_{QQ}^{N}(\omega)$ and $S_{QQ}^{N}(V)$ where, again no noise is added, but an extra screening charge is [9] for each edge state added, thus reducing the charge fluctuations seen in the gate, as more edge states are introduced.

5. Conclusions

In this work we have shown how to calculate the screened charge fluctuations in a gated N–S structure, we then proceeded to focus on the example of how charge fluctuations in the edge states of an N–S system differ from those of a normal system. The most startling difference is that for the example of a quantum point contact scatterer the screened charge fluctuations are unaffected by the presence of extra edge states. This is in contrast to the normal case [9]. This can be understood in the context of the electrochemical capacitance which for the N–S system is given by

$$C_\mu^{NS} = \frac{e^2NR_o^{NS}C}{C + e^2NR_o^{NS}}. \tag{25}$$

For perfect Andreev reflection $C_\mu^{NS} = 0$ in contrast to the normal system where the electrochemical capacitance is given by

$$C_\mu^{N} = \frac{e^2NC}{C + e^2N} \tag{26}$$

which for finite C and N implies $C_\mu^{N} \neq 0$. Comparing the two above equations we see that the introduction of Andreev reflection reduces the electrochemical capacitance. It has been shown elsewhere that fluctuations in the charge in normal structures, as characterized by R_q and R_V, determines the dephasing rate [9]. We expect that such a relationship is also true for normal superconducting hybrid structures. Finally induced currents have been observed in gates which are coupled to normal conductors [10]. Here we see, from equation (25), that charging is reduced in the presence of Andreev reflection and in the extreme case of perfect Andreev reflection the charging is zero.

This work was supported by the Swiss National Science Foundation and by the TMR network Dynamics of Nanostructures.

References

[1] Martin A M, Gramespacher T and Büttiker M *Phys. Rev.* B **60** R12581
[2] Takayanagi H and Akazaki T 1999 *Physica* B **251** 462
[3] Ma M and Zyuzin A Yu 1993 *Europhys. Lett.* **21** 941
[4] Ishikawa Y and Fukuyama H, 1999 *J. Phys. Soc. Japan* **68** 954
[5] Hoppe H, Zülicke U and Schön G *preprint* cond-mat/9909035
[6] Gramespacher T and Büttiker M *preprint* cond-mat/9908469
[7] Pedersen M H, van Langen S A and Büttiker M 1998 *Phys. Rev.* B **57** 1838
[8] Büttiker M 1996 *J. Math. Phys.* **37** 4793
[9] Büttiker M and Martin A M *Phys. Rev.* B *preprint* cond-mat/9902320
[10] Chen W, Smith III T P, Büttiker M, and Shayegan M 1994 *Phys. Rev. Lett.* **73** 146
Sommerfeld P K H, van der Heijden R W and Peeters F 1996 *Phys. Rev.* B **53** R13250